# Optimization:
# Stochastic Optmization

## Short Examples Series

## using

## Risk Simulator

**REAL Options Valuation**

***Optimization – Stochastic Portfolio Allocation***

**File Name:** *Optimization – Stochastic Portfolio Allocation*
**Location:** *Modeling Toolkit | Optimization | Stochastic Portfolio Allocation*
**Brief Description:** *This sample model illustrates how to run a stochastic optimization on continuous decision variables with simulation and interpreting optimization results*
**Requirements:** *Modeling Toolkit, Risk Simulator*

This model shows four asset classes with different risk and return characteristics. The idea here is to find the best portfolio allocation such that the portfolio's bang-for-the-buck or returns to risk ratio is maximized. That is, the goal is to allocate 100% of an individual's investment among several different asset classes (e.g., different types of mutual funds or investment styles: growth, value, aggressive growth, income, global, index, contrarian, momentum, and so forth). This model is different from others in that there exists several simulation assumptions (risk and return values for each asset), as seen in Figure 1.

A simulation is run, then optimization is executed, and the entire process is repeated multiple times to obtain distributions of each decision variable. The entire analysis can be automated using Stochastic Optimization.

In order to run an optimization, several key specifications on the model have to be identified first:

***Objective:*** *Maximize Return to Risk Ratio (C12)*
***Decision Variables:*** *Allocation Weights (E6:E9)*
***Restrictions on Decision Variables:*** *Minimum and Maximum Required (F6:G9)*
***Constraints:*** *Portfolio Total Allocation Weights 100% (E11 is set to 100%)*
***Simulation Assumptions:*** *Return and Risk Values (C6:D9)*

The model shows the various asset classes. Each asset class has its own set of annualized returns and annualized volatilities. These return and risk measures are annualized values such that they can be consistently compared across different asset classes. Returns are computed using the geometric average of the relative returns, while the risks are computed using the logarithmic relative stock returns approach.

The Allocation Weights in column E holds the decision variables, which are the variables that need to be tweaked and tested such that the total weight is constrained at 100% (cell E11). Typically, to start the optimization, we will set these cells to a uniform value. In this case, cells E6 to E9 are set at 25% each. In addition, each decision variable may have specific restrictions in its allowed range. In this example, the lower and upper allocations allowed are 10% and 40%, as seen in columns F and G. This setting means that each asset class may have its own allocation boundaries.
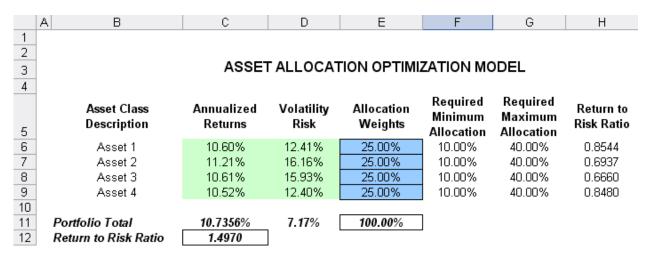
Figure 1: Asset allocation model ready for stochastic optimization

Next, column H shows the return to risk ratio, which is simply the return percentage divided by the risk percentage, where the higher this value, the higher the bang-for-the-buck. The remaining parts of the model show the individual asset class rankings by returns, risk, return to risk ratio, and allocation. In other words, these rankings show at a glance which asset class has the lowest risk, or the highest return, and so forth.

**Running an Optimization**

To run this model, simply click on **Risk Simulator | Optimization** | **Run Optimization**. Alternatively, and for practice, you can set up the model using the following steps.

1. Start a new profile (**Risk Simulator | New Profile**).

2. For stochastic optimization, set distributional assumptions on the risk and returns for each asset class. That is, select cell **C6** and set an assumption (**Risk Simulator | Set Input Assumption**) and make your own assumption as required. Repeat for cells **C7** to **D9**.

3. Select cell **E6**, and define the decision variable (**Risk Simulator | Optimization** | **Decision Variables** or click on the *Define Decision icon*) and make it a **Continuous Variable** and then link the decision variable's name and minimum/maximum required to the relevant cells (**B6, F6, G6**).

4. Then use the **Risk Simulator copy** on cell **E6**, select cells **E7 to E9**, and use **Risk Simulator's paste** (**Risk Simulator | Copy Parameter**) and **Risk Simulator | Paste Parameter** or use the copy and paste icons).
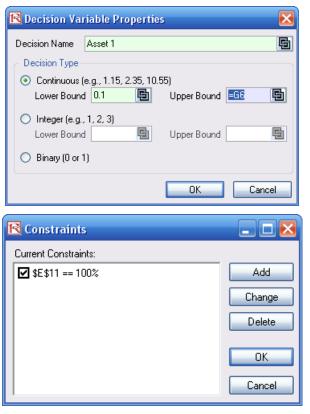
5. Next, set up the optimization's constraints by selecting **Risk Simulator | Optimization** | **Constraints**, selecting **ADD**, and selecting the cell **E11**, and making it equal **100%** (total allocation, and do not forget the % sign).

6. Select cell **C12**, the objective to be maximized and make it the objective: **Risk Simulator | Optimization** | **Set Objective** or click on the **O** icon.

7. Run the simulation by going to **Risk Simulator | Optimization** | **Run Optimization**. Review the different tabs to make sure that all the required inputs in steps 2 and 3 are correct. Select **Stochastic Optimization** and let it run for 500 trials repeated 20 times (Figure 2 illustrates these setup steps).

You may also try these other optimization routines:


*Discrete Optimization is an optimization that is run on a discrete or static model, where no simulations are run. This optimization type is applicable when the model is assumed to be known and no uncertainties exist. Also, a discrete optimization can be run first to determine the optimal portfolio and its corresponding optimal allocation of decision variables before more advanced optimization procedures are applied. For instance, before running a stochastic optimization problem, a discrete optimization is run to first determine if there exist solutions to the optimization problem before a more protracted analysis is performed.*

*Dynamic Optimization is applied when Monte Carlo simulation is used together with optimization. Another name for such a procedure is Simulation-Optimization. In other words, a simulation is run for N trials, and then an optimization process is run for M iterations until the optimal results are obtained or an infeasible set is found. Using Risk Simulator's optimization module, you can choose which forecast and assumption statistics to use and replace in the model after the simulation is run. Then you can apply these forecast statistics in the optimization process. This approach is useful when you have a large model with many interacting assumptions and forecasts, and when some of the forecast statistics are required in the optimization.*

*Stochastic Optimization is similar to the dynamic optimization procedure with the exception that the entire dynamic optimization process is repeated T times. The results will be a forecast chart of each decision variable with T values. In other words, a simulation is run and the forecast or assumption statistics are used in the optimization model to find the optimal allocation of decision variables. Then another simulation is run, generating different forecast statistics, and then these new updated values are optimized, and so forth. Hence, the final decision variables will each have its own forecast chart, indicating the range of optimal decision variables. For instance, instead of obtaining single-point estimates in the dynamic optimization procedure, you can now obtain a distribution of the decision variables, and, hence, a range of optimal values for each decision variable, also known as a stochastic optimization.*
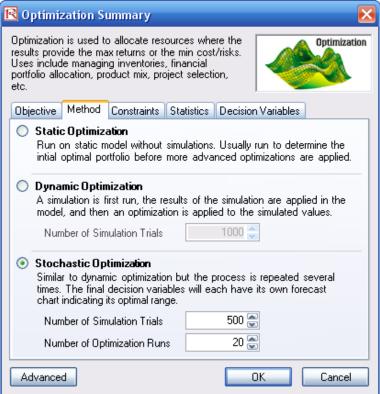
Figure 2: Setting up the stochastic optimization problem

**Viewing and Interpreting Forecast Results**

Stochastic optimization is performed when a simulation is first run and then the optimization is run. Then the whole analysis is repeated multiple times. The result is a distribution of each decision variable rather than a single-point estimate (Figure 3). This means that instead of saying you should invest 30.57% in Asset 1, the optimal decision is to invest between 30.10% and 30.99% as long as the total portfolio sums to 100%. This way, the results provide management or decision makers a range of flexibility in the optimal decisions. Refer to Chapter 11 of Modeling Risk: Applying Monte Carlo Simulation, Real Options Analysis, Forecasting, and Optimization by Dr. Johnathan Mun for more detailed explanations about this model, the different optimization techniques and an interpretation of the results. Chapter 11's appendix also details how the risk and return values are computed.
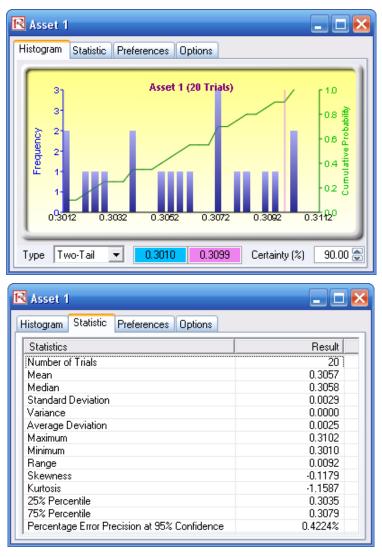


Figure 3: Simulated results from the stochastic optimization approach